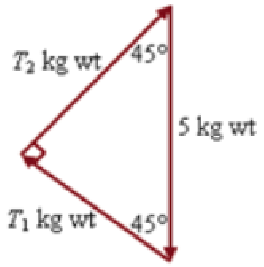
Exercise 20A: Solutions

1 $T_1 = 3$ kg wt
$T_2 = T_1 + 4 = 7$ kg wt

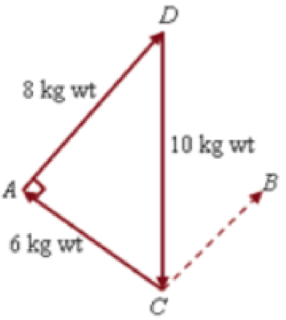
2 Rearrange into a triangle of forces.



Using trigonometry,

$$\begin{aligned} T_1 &= T_2 \\ &= 5 \sin 45^\circ \\ &= \frac{5\sqrt{2}}{2} \text{ kg wt} \end{aligned}$$

3 Rearrange into a triangle of forces.

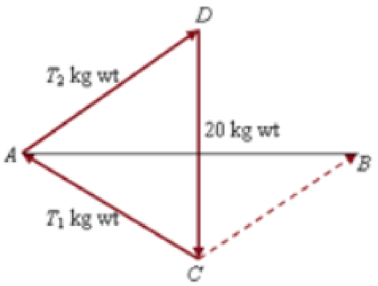


$\angle ACB = \angle ACD + \angle ADC$

These angles can be calculated using the cosine rule, but the student should notice that ΔACD is a 'doubled' 3- 4- 5 triangle with $\angle CAD = 90^\circ$.

$$\begin{aligned} \therefore ACB &= \angle ACD + \angle ADC \\ &= 180 - 90 = 90^\circ \end{aligned}$$

4 Rearrange into a triangle of forces.



Using the cosine rule in the triangle in the original diagram, it is clear that:

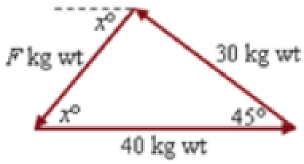
$$\begin{aligned} \cos \angle CAB &= \frac{15^2 + 10^2 - 12^2}{2 \times 15 \times 10} \\ &= 0.6033 \\ \angle\, CAB &= 52.89^\circ \\ \angle ADC &= 90 - \angle\, CAB \\ &= 37.11^\circ \\ \cos \angle CBA &= \frac{15^2 + 12^2 - 10^2}{2 \times 15 \times 12} \\ &= 0.7472 \end{aligned}$$

$$\angle CBA = 41.65^\circ$$
$$\angle ACD = 90 - \angle CBA$$
$$= 48.35^\circ$$
$$\angle CAD = 180 - 37.11 - 48.35$$
$$= 94.54^\circ$$

Use the sine rule to find T_1 and T_2.

$$\frac{T_1}{\sin \angle ACD} = \frac{20}{\sin \angle CAD}$$
$$T_1 = \frac{20 \times \sin 48.35^\circ}{\sin 94.54^\circ}$$
$$\approx 14.99 \text{ kg wt}$$
$$\frac{T_2}{\sin \angle ADC} = \frac{20}{\sin \angle CAD}$$
$$T_2 = \frac{20 \times \sin 37.11^\circ}{\sin 94.54^\circ}$$
$$\approx 12.10 \text{ kg wt}$$

5 Rearrange into a triangle of forces.



Using the cosine rule,

$$F^2 = 40^2 + 30^2 - 2 \times 30 \times 40 \times \cos 45^\circ$$
$$= 802.94$$
$$F \approx 28.34 \text{ kg wt}$$

Using the cosine rule,

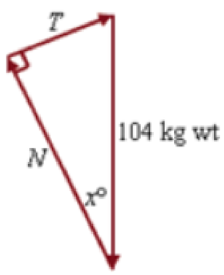
$$\cos x = \frac{F^2 + 40^2 - 30^2}{2 \times F \times 40}$$
$$= 0.663$$
$$x \approx 48.5^\circ$$

W 48.5° S or S 41.5° W

6 The angle between the plane and the horizontal is given by

$$\tan x = \frac{5}{12}$$
$$= 0.4167$$
$$x \approx 22.619^\circ$$

Rearrange into a triangle of forces.



$$T = 104 \sin x$$
$$= 40 \text{ kg wt}$$

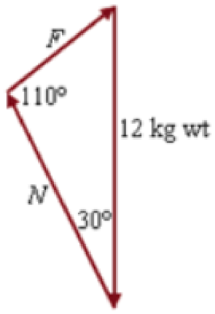
Note: The hypotenuse is 13, so

$\sin x = \dfrac{5}{13}$ and $\cos x = \dfrac{12}{13}$.

$$N = 104\cos x$$
$$= 96 \text{ kg wt}$$

7 Note that F will be acting at 50° to the horizontal and 70° to N, which becomes 110° when the force vectors joined head to tail.

Rearrange into a triangle of forces.

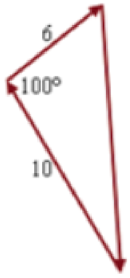


Use the sine rule.

$$\frac{F}{\sin 30^\circ} = \frac{12}{\sin 110^\circ}$$
$$F = \frac{12 \times \sin 30^\circ}{\sin 110^\circ} \approx 6.39 \text{ kg wt}$$

8 In each case, the particle will be in equilibrium if the forces add to zero. Draw the first two forces, and calculate the third force required for equilibrium.

a

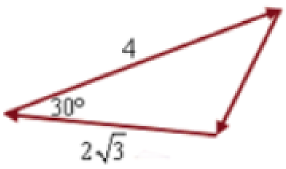


Use the cosine rule to calculate the magnitude of the third force.

$$F^2 = 10^2 + 6^2 - 2 \times 10 \times 6\cos 100^\circ$$
$$= 156.837$$
$$F \approx 12.52 \text{ kg wt}$$

This is not the force in the diagram, so these forces will not be in equilibrium.

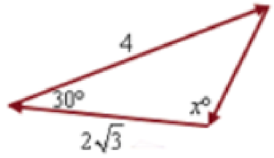
b



Use the cosine rule to calculate the magnitude of the third force.

$$F^2 = 4^2 + (2\sqrt{3})^2 - 2 \times 4$$
$$\times 2\sqrt{3} \times \cos 30^\circ$$
$$= 4$$
$$F = 2 \text{ kg wt}$$

It has the same magnitude as the third force in the diagram.



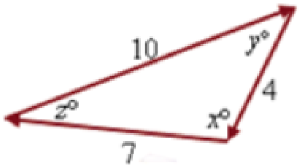
Use the sine rule to find x.

$$\frac{\sin x}{4} = \frac{\sin 30^\circ}{2}$$
$$\sin x = \frac{0.5 \times 4}{2} = 1$$
$$x = 90^\circ$$

This vector is at the same angle with the $2\sqrt{3}$ vector as in the original diagram.

$\therefore$ the vectors will be in equilibrium.

9 Draw the triangle of forces and use the cosine rule to find the three angles.
When the vectors are placed tail to tail, the angles between them will be the supplements of the angles in the triangle.



$$\cos x = \frac{7^2 + 4^2 - 10^2}{2 \times 7 \times 4}$$
$$= -0.625$$
$$x \approx 128.68^\circ$$

Angle between vectors is

$180^\circ - 128.68^\circ = 51.32^\circ$

$$\cos y = \frac{10^2 + 4^2 - 7^2}{2 \times 10 \times 4}$$
$$= 0.8375$$
$$y \approx 33.12^\circ$$

Angle between vectors is

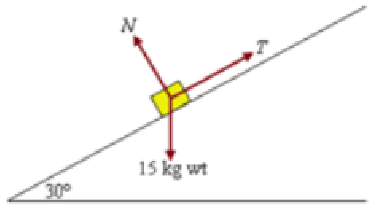
$180^\circ - 33.12^\circ = 146.88^\circ$

$$z \approx 180^\circ - 128.68^\circ - 33.12^\circ$$
$$= 18.2^\circ$$

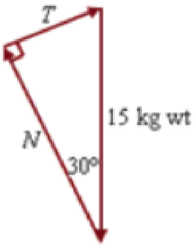
Angle between vectors is

$180^\circ - 18.2^\circ = 161.8^\circ$

10a



Draw the triangle of forces.

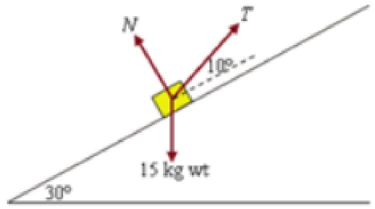


$$T = 15 \sin 30^\circ$$
$$= 7.5 \text{ kg wt}$$

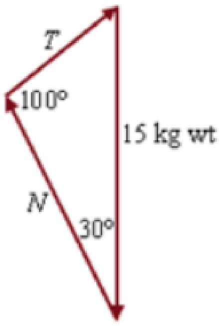
b The situation will be the same, except that the 30° angle will now be 40°.

$$T = 15 \sin 40^\circ$$
$$\approx 9.64 \text{ kg wt}$$

c The angle between T and N is now 80°.



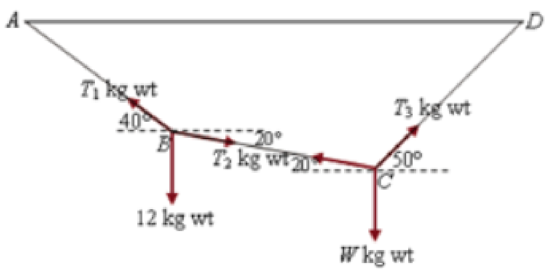
Draw the triangle of forces.



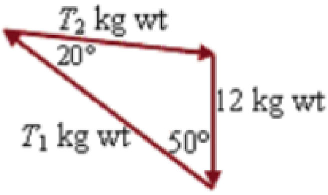
Use the sine rule.

$$\frac{T}{\sin 30^\circ} = \frac{15}{\sin 100^\circ}$$
$$T = \frac{15 \times 0.5}{\sin 100^\circ}$$
$$\approx 7.62 \text{ kg wt}$$

11



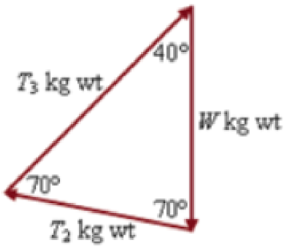
Draw the triangle of forces for point B.



Use the sine rule to find T_1 and T_2.

$$\frac{T_1}{\sin 110^\circ} = \frac{12}{\sin 20^\circ}$$
$$T_1 = \frac{12 \times \sin 110^\circ}{\sin 20^\circ}$$
$$\approx 32.97 \text{ kg wt}$$
$$\frac{T_2}{\sin 50^\circ} = \frac{12}{\sin 20^\circ}$$
$$T_2 = \frac{12 \times \sin 50^\circ}{\sin 20^\circ}$$
$$\approx 26.88 \text{ kg wt}$$

Now draw the triangle of forces for point C.



Use the sine rule to find T_3.

$$\frac{T_3}{\sin 70^\circ} = \frac{T_2}{\sin 40^\circ}$$
$$T_3 = \frac{26.88 \times \sin 70^\circ}{\sin 40^\circ}$$
$$\approx 39.29 \text{ kg wt}$$

Since the triangle is isosceles, $W = T_3 \approx 39.29 \text{ kg wt}$

The mass of W is 39.29 kg.